Pre-Calculus CP 1 – 3.1 – 3.3 I.C.E **Name:** KEY

For 1 and 2, write the exponential equation in logarithmic form.

1. $e^2 = 7.389...$

$\ln 7.389... = 2$

2. $9^{0.630...} = 4$

$\log_9 4 = 0.630...$

For 3 and 4, write the logarithmic equation in exponential form.

3. $\ln\left(\frac{2}{5}\right) = -0.916...$

$e^{-0.916...} = \frac{2}{5}$

4. $\log_7(9) = 1.129...$

$7^{1.129...} = 9$

For 5-10, use the One-to-One Property to solve the equation for x.

5. $2^{x+4} = 32$

$2^{x+4} = 2^5$

$x+4 = 5$

$x = 1$

6. $e^{7x+3} = e^{31}$

$7x+3 = 31$

$7x = 28$

$x = 4$

7. $\left(\frac{1}{5}\right)^{2x-1} = 625 = \left(\frac{1}{5}\right)^{-4}$

$2x-1 = -4$

$2x = -3$

$x = -3/2$

8. $\log(5x+3) = \log(12)$

$5x+3 = 12$

$5x = 9$

$x = 9/5$

9. $\log(x^3+3) = \log(11)$

$x^3+3 = 11$

$x^3 = 8$

$x = 2$

10. $\ln(2x+3) = \ln(x^2)$

$2x+3 = x^2$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3, -1$

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For 11-16, express in expanded form.

11. $\log_7(9x) =$ $\log_7 9 + \log_7 x$

12. $\log_3\left(\frac{y}{x}\right) =$ $\log_3 y - \log_3 x$

13. $\ln\sqrt{z} =$ $\frac{1}{2}\ln z$

14. $\ln\frac{x^3y^5}{\sqrt[3]{x+1}} =$ $\ln x^3y^5 - \frac{1}{3}\ln(x+1)$

$3\ln x + 5\ln y - \frac{1}{3}\ln(x+1)$

15. $\log_3(3x)^3 =$ $3(\log_3 3 + \log_3 x)$

$= 3(1 + \log_3 x) = 3 + 3\log_3 x$

16. $\ln\sqrt{\frac{x^2}{y^3}} =$ $\frac{1}{2}(2\ln x - 3\ln y)$

$= \ln x - \frac{3}{2}\ln y$

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For 17-19, condense the expression into a single logarithm.

17. $2\ln 8+5\ln(z-4)=$ $\ln 8^2+\ln(z-4)^5$
$=\ln[64(z-4)^5]$

18. $5\log_3 2x-\log_3 9y=$ $\log_3\frac{(2x)^5}{9y}=\log_3\frac{32x^5}{9y}$

19. $\frac{1}{3}\left[2\log(x+3)+\log x-\log(x^2-1)\right]=$ $\log\sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$

For 20-22, evaluate the logarithm **without using a calculator.**

20. $\log_4 32-\log_4 2+\log_4 256=$ $\log_4 2^5-\log_4 2+\log_4 2^8$
$=\log_4(2^2)^{\frac{5}{2}}-\log_4(2^2)^{\frac{1}{2}}+\log_4(2^2)^4$
$=5/2-1/2+4=6$

21. $2\ln e^6-\ln e^5=$ $2(6)-5=7$

22. $3\log_3 27-\log_3 9=$ $3\log_3 3^3-\log_3 3^2$
$=3(3)-2=7$

Word Problems

23. A total of $10,000 was invested at an annual interest rate of 7.5%. What is the balance after 10 years if it is compounded

a) monthly

$A = 10{,}000\left(1 + \frac{.075}{12}\right)^{12 \cdot 10}$

$= \$21{,}120.65$

b) quarterly

$A = 10{,}000\left(1 + \frac{.075}{4}\right)^{4 \cdot 10}$

$= \$21{,}023.49$

c) continuously

$A = Pe^{rt} = 10{,}000\, e^{(.075)10}$

$= \$21{,}170$

24. The population of a town increases according to the model $P(t) = 2500e^{0.2197t}$, where t is the time in years, with t = 0 corresponding to 2000. Use the model to estimate the population in

a) 2006

$P = 2500e^{0.2197(6)} = 9{,}342$

b) present day

$P(15) = 2500\, e^{0.2197(15)} = 67{,}477$

25) Radioactive Decay

In 1986, a scientist built a time machine out of a car to travel back in time. While experimenting with his time machine, an explosion spread plutonium, the key chemical in his invention, over hundreds of square miles. To figure out why the land is still uninhabited, consider the model

$$P=10\left(\frac{1}{2}\right)^{t/24{,}100}$$

This equation represents the amount of plutonium, P, after t years. If the initial amount of plutonium was 10 pounds and t = 0 represents 1986, how much will remain in the year 2009?

$t=23$

$P=10\left(\frac{1}{2}\right)^{\frac{23}{24,100}} = 9.99$ lbs !

26) Human Memory Model

Students participating in a psychology experiment attended several lectures on a subject and were given an exam. Every month for a year after the exam, the students were retested to see how much of the material they remembered. The average scores for the group are given by the human memory model:

$$f(t)=75-6\ln(t+1),\quad 0\le t\le 12$$

where t is the time in months.

a) What was the average score on the original (t = 0) exam?

$f(0) = 75 - 6\ln(0+1) = 75$

b) What was the average score at the end of 2 months?

$f(2) = 75 - 6\ln(2+1) = 68.41$

c) What was the average score at the end of 6 months?

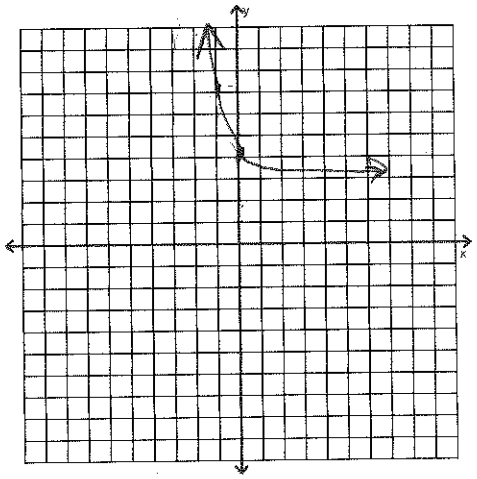
$f(6) = 75 - 6\ln(6+1) = 63.32$

Graphing

Sketch graphs of the following exponentials and logarithms based on the parent function and the transformation:

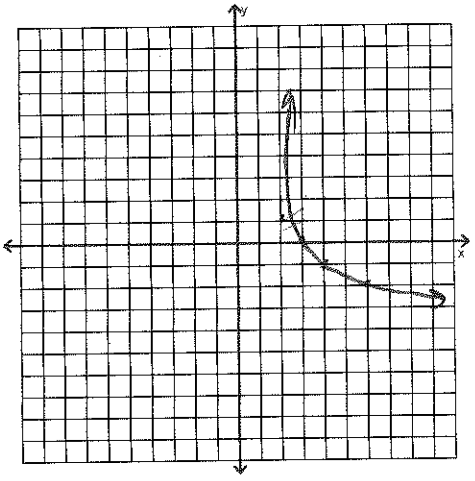
a) $y = 2^{-x} + 3$

reflect over y-axis
shift up 3



b) $y = -\log_2(x-2)$

reflect over x-axis
shift right 2



c) $y = -\left(\frac{1}{3}\right)^{x-1} - 2$

$y = \left(\frac{1}{3}\right)^x$

reflect over x
shift right 1
shift 2 down